Analysis the trapping electron by left circular polarization electromagnetic wave in static electric field

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Introduction

The

The interaction of electrons with electromagnetic waves in the presence of external static electric and magnetic fields represents a fundamental problem in plasma physics and electromagnetic theory with profound implications for both basic science and technological applications [1,2]. This complex multi-field environment gives rise to rich nonlinear dynamics that govern phenomena ranging from particle acceleration in astrophysical plasmas to the operation of microwave devices and free-electron lasers [3].

The method is inspired by Ref. [1], where the electron motion under electromagnetic wave is analyzed. Here the static electric field is considered.

**Field and particle equations**

Assume the wave vector is parallel to the uniform background magnetic field in the z direction and assume the phase velocity as required for anomalous doppler effect. The ratio of the strength of the wave magnetic field to the background magnetic field is defined as , so the total magnetic field can be written as . The static electric field has strength E0 in z axis and the total electric field is where the wave electric field with left-hand circular polarization is

Faraday’s law requires the associated magnetic field to be

The charge particle move equation is determined by the relativistic Lorentz equation

Where and

**Transformation to the wave frame**

In wave frame, which denotes as prime and moves at constant velocity with respect to the lab frame, the fields are

Where . Substitute \*\* into \*\* we have and

Since and {**x**, ict} are four-vectors, than we have

where is the wavenumber in the wave frame. The wave magnetic field is than

the motion equation of the charge particle in the prime frame is

Where . Note that ,, and differ and should not be confused with each other

Here is the cyclotron frequency of a nonrelativistic particle in the lab frame, according to eq.(12) , eq.(13), we have

Combine with eq.(9) , we have

**Construction of pseudo-potential problem**

Taking the derivative of Eq. (14) with respect to t’ gives

Combing with eq.(9), the time derivation of wave magnetic field in wave frame is

where here .Substitution using eq. (16), eq.(17) and eq.( 19) into eq.(18) gives

Reorganize the equations, we have

Ignoring the high order of terms gives

here . Define the

The actually is frequency mismatch if the resonant condition is , here let’s prove it :

Since {} are four-vector, we have

Substituting eq. (26) into gives

The actually is nonlinear function of , which is different with no static electric field as in paper  [1],since

To express approximately linearly with , one must have to

To find the linear range of the approximation, the ratio of two part must have

Since ,we have

We have when . In the condition as , we can confidently say that is linear with .

**How to** **connect with ?**

To obtain such expression, insert eq. (17) and eq. (19) into the time derivation of gives

Inserting eq. (33) and eq. (34) into eq. (32) gives

Here the equation is first-order linear differential equation of the form:

The solution of z is

here and , . Since P << 1, we have

The eq. (35) can ignore the second term and written as:

The substitution of eq. (22) into eq. (38) gives

Integrating eq. (39) with and ignoring the second term on the right sight gives:

Noting that t = 0 corresponds to z’=0 because z = 0 at t= 0 and recalling the four vectors {}, it has relation that

Since is in the y direction when z’ = 0, it is seen that

Where is defined by and , . With these definitions, eq. (40) becomes

The substitution of eq. (42) into eq. (24) gives

Simplifying eq. (44) and substituting for gives:

Here sig(q) is the sign of charge q, if q>0, sig(q) =1, else sig(q)=-1, beside this,. Substituting eq. (46) into eq. (45) gives

Where

Multiplying eq. (47) by and integrating gives a pseudo-energy equation

and

Where ,, The term with E0 can only be solved numerically.

When E0=0, we have

From eq. (16) and , we see that

At the start where t = 0 and z = 0,

**Solve**

According to eq. (15) , multiplying on both sides gives

Ignoring third term on the right side and reorganize the equation, we have

According to eq. (14), multiplying on both sides gives

Add eq. (58) and eq. (57) , we have

Which means energy change ratio equal to work done by static electric field.

Integrating both side with t’ gives

**Calculate**

According to eq. (16), the first term on the right sight is approximately expressed as

According to eq. (60), the second term on the right sight is given as

define , we have

Finally, we have

This try end in failure .

